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EIGENFUNCTION LOCALIZATION IN THE QUANTIZED RIGID BODY

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1. Introduction

It has long been known [5],[10],[12],[16],[17] that given a stable, closed, elliptic geodesic γ, one can associate with this curve a sequence of quasimodes ϕ_n for the corresponding Laplace operator $-\Delta$, in the sense that the ϕ_n have microsupport in a tube of width $\mathcal{O}(n^{-1/2})$ about γ and decay exponentially outside this tube. On the other hand, in the unstable, hyperbolic case, it is known [13] that under suitable hypotheses, one can associate complex resonances with hyperbolic orbits. However, analogous general results are not known for eigenvalues and eigenfunctions (see, however [4],[6],[7]. In this paper we focus on a specific paradigm; namely, that of the asymmetric rigid body reduced at an S^1 Noether symmetry. The corresponding quantum system on S^2 is integrable with the classical Lamé harmonics as joint eigenfunctions [20]. The classical system inherits a natural hyperbolic geodesic Γ corresponding to the unstable rotation about the middle-length inertial axis. Given the quantum Hamiltonian $\mathcal{H}$, we show that there is a sequence of L^2-normalized eigenfunctions, ψ_n, with L^∞ norm concentrated along Γ. More precisely, let $\Gamma(n^{-1})$ denote a tube of width $\mathcal{O}(n^{-1})$ about Γ, and let $V_j; j = 1,2,3,4$ denote arbitrarily small (but fixed) disconnected neighbourhoods about the four umbilic points on Γ. Our main results are:

$$\|\psi_n\|_{L^\infty(\Gamma(n^{-1})-\bigcup_j V_j)} = C\frac{n^{1/4}}{\log n} + \mathcal{O}\left(\frac{n^{1/4}}{(\log n)^2}\right),$$

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