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CURVATURE VARIFOLDS WITH BOUNDARY

CARLO MANTEGAZZA

Abstract

We introduce a new class of nonoriented sets in $\mathbb{R}^k$ endowed with a generalized notion of the second fundamental form and boundary, proving several compactness and structure properties. Our work extends the definition and some results of J. E. Hutchinson [25] and can be applied to variational problems involving surfaces with boundary.

1. Introduction

Some problems in the calculus of variations are concerned with the existence of minima for functionals defined on smooth manifolds embedded in $\mathbb{R}^k$ and involving quantities related to the geometry of the manifolds. The functionals which we are interested in depend on the curvature tensor of the manifolds. As usual, in order to get the existence of minimizers by the so called direct methods of calculus of variations it is necessary to enlarge the space where the functional is defined and to work out a compactness–semicontinuity theorem in the enlarged domain.

The aim of this paper is to introduce a new class of n–dimensional sets endowed with a weak notion of the second fundamental form and boundary. We prove that this class has good compactness and structure properties.

Our work is based on the theory of integer rectifiable varifolds developed by Allard in [15], [16] (see section 2). Roughly speaking, an integer n–varifold is an n–dimensional set in $\mathbb{R}^k$ endowed with an integer multiplicity; smooth n–dimensional manifolds can be considered as unit density varifolds.

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