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NON-ZERO DEGREE MAPS AND SURFACE BUNDLES OVER S^1

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1. Introduction

In this paper we study non-zero degree maps $f : M \to N$ between 3-dimensional compact orientable irreducible ∂-irreducible manifolds.

An important question in topology is to decide whether there exists a map of non-zero degree between given manifolds of the same dimension. One can think of the existence of such a map as defining a partial ordering on the set of homeomorphic classes of compact connected manifolds of a given dimension. As suggested by M.Gromov, this partial order can be defined as follows: say that M dominates N, denoted by $M \geq N$, if there is a non-zero degree allowable map from M to N. From H.C.Wang's theorem and Gromov's work [18, Chap 6], it follows that each closed hyperbolic orientable n-manifolds with $n \neq 3$ dominates only finitely many closed orientable hyperbolic n-manifolds. We show that this result fails in dimension 3. This was first established by the second author.

In the case of surface bundles over S^1 we show that if a bundle over S^1 dominates an irreducible ∂-irreducible 3-manifold N, then either the first Betti number decreases or N is a bundle over S^1. Moreover using Thurston's norm on $H^1(., R)$, we give a necessary and sufficient condition for such maps to be homotopic to a covering or a homeomorphism.

We apply those facts to study W.Thurston's conjecture which claims that any complete finite volume hyperbolic 3-manifold is finitely covered by a surface bundle over S^1. We prove that for any integer $n > 0$, there are infinitely many closed hyperbolic orientable 3-manifolds with first Betti number n such that no tower of abelian coverings over M contains a surface fiber bundle over S^1. So if Thurston's conjecture is true, the coverings involved must be much more complicated than towers of

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