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FREE KLEINIAN GROUPS AND VOLUMES OF HYPERBOLIC 3-MANIFOLDS

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1. Introduction

The central result of this paper, Theorem 6.1, gives a constraint that must be satisfied by the generators of any free, topologically tame Kleinian group without parabolic elements. The following result is case (a) of Theorem 6.1.

Main Theorem. *Let $k \geq 2$ be an integer and let Φ be a purely loxodromic, topologically tame discrete subgroup of* $\mathrm{Isom}_+(\mathbf{H}^3)$ *which is freely generated by elements $\xi_1, \dots, \xi_k$. Let z be any point of* $\mathbf{H}^3$ *and set* $d_i = \mathrm{dist}(z, \xi_i \cdot z)$ *for $i = 1, \dots, k$. Then we have*

$$\sum_{i=1}^{k} \frac{1}{1+e^{d_i}} \leq \frac{1}{2}.$$

In particular there is some $i \in \{1, \dots, k\}$ such that $d_i \geq \log(2k-1)$.

The last sentence of the Main Theorem, in the case $k = 2$, is equivalent to the main theorem of [14]. While most of the work in proving this generalization involves the extension from rank 2 to higher ranks, the main conclusion above is strictly stronger than the main theorem of [14] even in the case $k = 2$.

Like the main result of [14], Theorem 6.1 has applications to the study of large classes of hyperbolic 3-manifolds. This is because many subgroups of the fundamental groups of such manifolds can be shown to be free by topological arguments. The constraints on these free subgroups

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