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ALGEBRAIC SOLUTIONS OF ONE-DIMENSIONAL FOLIATIONS

A. LINS NETO & M.G. SOARES

Abstract

In this article we consider the problem of extending the result of J.P.Jouanolou on the density of singular holomorphic foliations on $\mathbf{C}P(2)$ without algebraic solutions to the case of foliations by curves of $\mathbf{C}P(n)$.

1. Introduction and statement of results

A one-dimensional (singular) holomorphic foliation $\mathcal{F}$ on $\mathbf{C}P(n)$ is given by a morphism

$$\Upsilon : \mathcal{O}(-d) \longrightarrow T\mathbf{C}P(n)$$

with singular set $sing(\mathcal{F}) = \{p : \Upsilon(p) = 0\}$. We will consider foliations with singular set in codimension greater than 1. Such a foliation $\mathcal{F}$ is represented in affine coordinates $(x_1, \ldots, x_n)$ by a vector field of the form

$$X = gR + \sum_{\ell=0}^{d} X_\ell$$

where R is the radial vector field $R = \sum_{i=1}^{n} x_i \frac{\partial}{\partial x_i}$, g is a homogeneous polynomial of degree d and X_ℓ is a vector field whose components are homogeneous polynomials of degree ℓ, $0 \leq \ell \leq d$. Since $sing(\mathcal{F})$ has codimension greater than 1 we have either $g \not\equiv 0$ or $g \equiv 0$ and X_d cannot be written as hR where h is homogeneous of degree $d-1$. In this case X has a pole of order $d-1$ at infinity (see [6]). We call d the *degree* of the foliation.

If $\mathcal{F}$ is a holomorphic foliation of dimension 1 on $\mathbf{C}P(n)$ with singular set $sing(\mathcal{F})$ and $\Gamma \subset \mathbf{C}P(n)$ is an irreducible algebraic curve, we say that Γ is an *algebraic solution* of $\mathcal{F}$ provided $\Gamma \setminus sing(\mathcal{F})$ is a leaf of the foliation. We prove the following :

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