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CLOSED WEINGARTEN HYPERSURFACES IN RIEMANNIAN MANIFOLDS

CLAUS GERHARDT

0. Introduction

In a complete $(n+1)$-dimensional manifold N we want to find closed hypersurfaces M of *prescribed curvature*, so-called *Weingarten* hypersurfaces. To be more precise, let Ω be a connected open subset of N, $f \in C^{2,\alpha}(\bar{\Omega})$, F a smooth, symmetric function defined in the positive cone $\Gamma_+ \subset \mathbf{R}^n$. Then we look for a convex hypersurface $M \subset \Omega$ such that

$$F|_M = f(x) \quad \forall x \in M, \tag{0.1}$$

where $F|_M$ means that F is evaluated at the vector $(\kappa_i(x))$ the components of which are the principal curvatures of M.

This is in general a problem for a fully nonlinear partial differential equation, which is elliptic if we assume F to satisfy

$$\frac{\partial F}{\partial \kappa_i} > 0 \quad \text{in} \quad \Gamma_+. \tag{0.2}$$

Classical examples of curvature functions F are the elementary symmetric polynomials H_k of order k defined by

$$H_k = \sum_{i_1 < \ldots < i_k} \kappa_{i_1} \ldots \kappa_{i_k}, \quad 1 \le k \le n. \tag{0.3}$$

H_1 is the mean curvature H, H_2 is the scalar curvature - for hypersurfaces in Euclidean space -, and H_n is the Gaussian curvature K.

For technical reasons it is convenient to consider, instead of H_k, the homogeneous polynomials of degree 1

$$\sigma_k = H_k^{1/k}, \tag{0.4}$$

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