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DISCRETE SURFACES WITH CONSTANT NEGATIVE GAUSSIAN CURVATURE AND THE HIROTA EQUATION

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1. Introduction

Surfaces with constant curvature (especially, with constant negative curvature $K = -1$, which we will call shortly K-surfaces) were one of the favorite objects of investigation in differential geometry in the 19th century (see [2], [10]). During this classical period many properties, which nowadays might be called *integrable* were discovered and many explicit examples of surfaces were constructed.

Later on the fashion of constructing explicit examples changed to proving that certain examples do not exist. The structure of the spaces of surfaces with constant curvature was partially clarified. Typical examples of the results were theorems proving that the only surface satisfying some prescribed assumptions is a round sphere.

A modern period of interest in this theory started with the paper [24] by Wente, where the simplest tori with constant mean curvature (abbreviated to CMC) were constructed. This turned out to be an interesting alternative to the theorems mentioned above. Further progress is mostly due to the theory of integrable equations - theory of solitons, which appeared in 1960's. Though this theory was oriented basically towards problems of mathematical physics, it deals in many cases with the same equations as differential geometry. The sine-Gordon equation

$$\phi_{xt} - \sin\phi = 0, \tag{1.1}$$

which is the Gauss equation for the K-surfaces, is one of the fundamental examples in this theory [11]. A characteristic result obtained with

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