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ON THE RESIDUE OF THE SPECTRAL ZETA FUNCTIONS OF KÄHLER MANIFOLDS WITH CONICAL SINGULARITIES

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0. Introduction

Let $\pi : M \to B$ be a family of Kähler manifolds, and $p : \xi \to M$ a holomorphic vector bundle with a Hermitian metric. Then, from the work of Quillen, the Knudsen-Mumford determinant $\lambda(\xi)$ admits a canonical Hermitian metric called the Quillen metric. In [9], [10], [11], Bismut, Gillet and Soulé calculated the curvature of $\lambda(\xi)$ and obtained the refinement of Grothendieck-Riemann-Roch theorem. In [3], their result was generalized to the case of degenerating family of Riemman surfaces by Bismut and Bost. But there is no result on the curvature of Knudsen-Mumford determinant for family of Kähler manifolds with boundary or singular Kähler manifolds.

As for the real case, in [4] - [6], Bismut and Cheeger extended the result of Atiyah-Patodi-Singer on the index of the Dirac operator on manifolds with boundary. They patched a cone to the boundary of the manifold and considered a manifold with conical singularities. They gave a detailed study of elliptic operators on such singular manifolds and obtained the family index theorem.

To consider the extension of the formula in [4], [5] in the direction of [7], [8] and [9] - [11], it is necessary to define the Quillen metric for the family of manifolds with conical singularities. Therefore we must consider the Ray-Singer analytic torsion on manifolds with conical singularities. By definition, it is given by a certain sum of the derivative at the origin of spectral zeta functions. From the results of Cheeger, these zeta functions possibly have a simple pole at the origin. Thus it is not clear whether the analytic torsion is defined for them.

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