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VANISHING THEOREMS FOR COHOMOLOGIES OF AUTOMORPHIC VECTOR BUNDLES

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1. Introduction

Let M be a compact, irreducible, locally hermitian symmetric space of noncompact type. If M is not a Riemann surface, then Calabi and Vesentini have shown in ([4]), that the complex structure on M is infinitesimally rigid, i.e., they show the vanishing of $H^1(M, \Theta_M)$, where Θ_M is the tangent sheaf of germs of holomorphic vector fields on M. Their method involves the construction of an 'auxiliary expression', which is simplified in two different ways, to obtain a quadratic form involving curvature terms. The desired vanishing is reduced then to proving that the quadratic form is positive definite. One obtains criteria for the vanishing of the cohomology groups $H^*(M, \Theta_M)$, which depend on the curvature properties of M and not on the lattice defining M.

Based on their method, Weil showed (see [17]) that an irreducible, cocompact lattice Γ in a real semisimple Lie group G without compact or three dimensional factors is rigid, i.e., any 'nearby' deformations of Γ inside G are equivalent. This amounts to showing the vanishing of $H^1(\Gamma, Ad)$, where Ad is the adjoint representation of G on its Lie algebra. Matsushima refined this method to show vanishing of Betti numbers of M below some degree ([10]).

Our main result is to give a criterion for the vanishing of cohomologies of "automorphic" vector bundles, generalising the results of Calabi-Vesentini and Matsushima. More generally we give a criterion for the vanishing of $\bar{\partial}$-cohomology (or $(\underline{q}, K^{\mathbf{C}})$–cohomology) of unitary $\underline{g}$- modules with coeffecients in a K- module. The terms involved can be calculated explicitly in terms of the dominant weight of the reprsentation defining the automorphic vector bundle and the curva-

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