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COMPACT RIEMANNIAN 7-MANIFOLDS WITH HOLONOMY G_2. II

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1. Introduction

This is the second of two papers about metrics of holonomy G_2 on compact 7-manifolds. In our first paper [15] we established the existence of a family of metrics of holonomy G_2 on a single, compact, simply-connected 7-manifold M, using three general results, Theorems A, B and C. Our purpose in this paper is to explore the theory of compact riemannian 7-manifolds with holonomy G_2 in greater detail. By relying on Theorems A-C we will be able to avoid the emphasis on analysis that characterized [15], so that this sequel will have a more topological flavour.

The paper has four chapters. The first chapter consists of introductory material. Section 1.1 gives some elementary geometric and topological material on compact 7-manifolds with torsion-free G_2- structures. Then §1.2 describes the holonomy groups $SU(2)$ and $SU(3)$, and §1.3 explains the concept of asymptotically locally Euclidean riemannian manifolds (shortened to ALE spaces) with special holonomy.

Recall that in [15], a compact 7-manifold M was defined by desingularizing a quotient T^7/Γ of the 7-torus by a finite group of isometries $\Gamma \cong \mathbb{Z}_2^3$. The subject of Chapters 2 and 3 is a generalization of this idea. Chapter 2 defines a general construction for compact 7-manifolds with torsion-free G_2- structures, which works by desingularizing quotients T^7/Γ for finite groups Γ. The ALE spaces with holonomy $SU(2)$ and $SU(3)$ discussed in §1.3 are an essential ingredient in performing this desingularization.

The central result of Chapter 2 is Theorem 2.2.3, which states that given a suitable finite group Γ and certain other data, one may con-

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