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COMPACT RIEMANNIAN 7-MANIFOLDS WITH HOLONOMY G_2. I

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1. Introduction

The list of possible holonomy groups of Riemannian manifolds given by Berger [3] includes three intriguing special cases, the holonomy groups G_2, Spin(7) and Spin(9) in dimensions 7, 8 and 16 respectively. Subsequently [1] it was shown that Spin(9) does not occur as a non-symmetric holonomy group, but Bryant [5] showed that both G_2 and Spin(7) do occur as non-symmetric holonomy groups. Bryant's proof is a local one, in that it proves the existence of many metrics of holonomy G_2 and Spin(7) on small balls in $\mathbb{R}^7$ and $\mathbb{R}^8$ respectively. He also gives some explicit examples of such metrics. In a subsequent paper [6], Bryant and Salamon construct *complete* metrics of holonomy G_2.

This is the first of two papers in which we shall construct examples of *compact* riemannian 7-manifolds with holonomy G_2. These are, to our knowledge, the first such examples known. We believe that they are the first nontrivial examples of odd-dimensional, compact, Ricci-flat riemannian manifolds. The author has also used similar methods to construct compact 8-manifolds with holonomy Spin(7), [12].

The goal of this first paper is to study a single example in depth. We shall describe a certain simply-connected, compact 7-manifold M, and construct a family of metrics on it with holonomy G_2. The 7-manifold M was chosen because it is the simplest example that we know of. The content of the paper is mostly introductory material, and proofs using a lot of analysis. The second paper will describe many different compact 7-manifolds admitting metrics of holonomy G_2, and will have a more topological emphasis. It will also contain much more discussion of the results, and some interesting questions.

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