J.DIFFERENTIAL GEOMETRY
Vol. 43, No.2 March, 1996

DETERMINANTAL FORMULAS FOR ORTHOGONAL AND SYMPLECTIC DEGENERACY LOCI

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Abstract

Given a vector bundle V of rank n on a variety X, together with two complete flags of subbundles, there is a degeneracy locus $X_w \subset X$ for each w in the symmetric group S_n. With suitable genericity hypotheses, the class of X_w in the Chow group of X is given by a double Schubert polynomial in the first Chern classes of the quotient line bundles of the flags [9]. In this note we give similar formulas for corresponding loci when V has an orthogonal or symplectic structure and the flags are isotropic; there is one such locus X_w for each w in the corresponding Weyl group.

Introduction

For any partition $\lambda = (\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_k > 0)$, and a formal sum $c_0 + c_1 + c_2 + ...$ of commuting elements c_i in a ring, with $c_i = 0$ if $i < 0$, denote by $\Delta_\lambda(c)$ the "Schur determinant"

$$\Delta_\lambda(c) = \det(c_{\lambda_i+j-i})_{1\leq i,j\leq k}.$$

The partition $(k, k-1, \dots, 1)$ whose i^{th} term is $k+1-i$ will be denoted by $\rho(k)$. In this introduction, for simplicity, we consider degeneracy loci on a nonsingular ambient variety X, with the assumption that the maps are generic enough so that all the loci have the expected codimension. We postpone to the next section the precise description of these loci as subschemes, and the statements without the assumptions of smoothness or genericity. In this introduction, formulas for these loci are given in the Chow rings with rational coefficients, but this can be improved to the Chow rings with integer coefficients.

Suppose V is a vector bundle of rank n on a nonsingular algebraic variety X, and E and F are subbundles of ranks e and f.

For $k \leq \min(e, f)$, let D_k be the locus in X where the dimension of the intersection of the fibers $E(x) \cap F(x)$ is at least k. One form of the Giambelli-Thom-Porteous formula states that if D_k has the expected

Received February 3, 1994 and, in revised form, October 31, 1994.