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STABILITY OF COMPLEX VECTOR BUNDLES

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0. Introduction

The notion of stability plays a central role in complex and algebraic geometry.

It was introduced by D. Mumford [5] and F. Takemoto [10] for the study of the moduli space of holomorphic vector bundles; S. Kobayshi and M. Lübke found that for irreducible bundles the existence of a Hermitian-Einstein metric is a sufficient condition for stability, and a major achievement of the theory has consisted in the work of M. Narashimhan and C. Seshadri for algebraic curves, S. Donaldson in the case of algebraic manifolds, K. Uhlenbeck and S.T. Yau for general Kähler manifolds (easily extended to regularized Hermitian n-manifolds, i.e., whose Kähler form η satisfies $\partial\overline{\partial}\eta^{n-1} = 0$) proving the existence of a Hermitian-Einstein connection on stable holomorphic vector bundles ([6], [1], [12]). Further generalization to Higgs bundles can be found in [2] and [9].

These results have made the tools of differential geometry available to complex and algebraic geometry, leading to several important applications, e.g., a much more extensive comprehension of Bogomolov-Gieseker type inequalities and the characterization of flat vector bundles. On the other hand, a general theory of the existence of holomorphic structures on complex bundles is far from being understood, and therefore it is very natural to try to extend the differential geometric characterization of stability to complex bundles with an unnecessarily integrable almost complex structure.

The first main result of the present paper is the following.

Theorem 0.1. *Assume a complex vector bundle over a compact almost Hermitian regularized manifold is equipped with a stable almost complex structure. Then it admits a Hermitian-Einstein connection.*

The notion of stability which we consider is the following: we require that $\mu(F) < \mu(E)$ holds for any J-holomorfic subbundle $F \subset E$ which

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