

## ON THE MOD $k$ INDEX THEOREM OF FREED AND MELROSE

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The purpose of this short note is to present an alternative approach to a formula of Freed-Melrose [6, Corollary 5.4], which expresses the topological index of vector bundles over  $Z/k$ -manifolds through geometric data.

Recall that Freed and Melrose proved their formula by first establishing a general index theorem for  $Z/k$ -manifolds and then making an application of the Atiyah-Patodi-Singer index theorem for manifolds with boundary [2].

Our approach is based on a result established jointly by Bismut and the author in [4] concerning the behaviour of the  $\eta$ -invariants under real embeddings. By such approach the use of the Atiyah-Patodi-Singer index theorem mentioned above is avoided. From our argument, it turns out immediately that for certain special dimensions, one can refine the  $Z/k$  index formula to a  $2Z/2k$  formula. Furthermore, our method also suggests a promised new approach to the Atiyah-Patodi-Singer index theorem itself.

This paper is organized as follows. In Section 1, we recall the basic notation and facts about  $Z/k$ -manifolds. In Section 2, we give our approach to the Freed-Melrose formula in which we are interested. Section 3 contains a  $2Z/2k$  refinement for dimension  $8k + 4$ . In the final Section 4, we discuss the Atiyah-Patodi-Singer index theorem for manifolds with boundary [2] from the point of view of our approach.

### 1. The topological index for $Z/k$ -manifolds

$Z/k$ -manifolds were introduced by Sullivan in his studies of geometric

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