J. DIFFERENTIAL GEOMETRY
Vol. 43, No. 1 January, 1996

DEHN SURGERY ON ARBORESCENT KNOTS

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1. Introduction

A knot K is called an *arborescent knot* if it can be obtained by summing and gluing several rational tangles together; see [7] or below for more detailed definitions. Recall that a 3-manifold is called a Haken manifold if it is irreducible and contains an incompressible surface. Following Hatcher [14] we say that a 3-manifold M is *laminar* if it contains an essential lamination. The purpose of this paper is to study Dehn surgeries on arborescent knots, and to see which of these surgered manifolds are laminar, Haken, or hyperbolic.

There has been some study on these problems for Montesinos knots. Denote by $K = K(p_1/q_1, \dots, p_n/q_n)$ a Montesinos knot obtained by gluing rational tangles corresponding to the rational numbers p_i/q_i together in a cyclic way; see for example [24] for more details. To avoid the trivial case, we always assume that $|q_i| \geq 2$. We call n the length of K. Oertel [24] showed that if $n \leq 3$, then there are no closed essential surfaces in the knot exterior $E(K) = S^3 - \operatorname{Int} N(K)$, and if $n \geq 4$ and $|q_i| \geq 3$, then there are incompressible surfaces which remain incompressible after all nontrivial surgeries. Delman [4], [5] studied essential laminations in $E(K)$, the exterior of K, showing that for most Montesinos knots there are essential laminations in $E(K)$ which remain essential after all nontrivial surgeries. The result is particularly interesting for those K with $n \leq 3$, because by the results of Oertel [24] and Hatcher [13] most of these surgered manifolds are nonHaken manifolds.

For our purpose we divide arborescent knots into three types. Type I knots are those Montesinos knots which have length at most 3. A knot is of type II if it is of the form shown in Figure 1.1, where $R(p_i/q_i)$ are rational tangles with $|q_i| \geq 2$, and B is any 4-string braid from the left

Received April 12, 1994.