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SUBMANIFOLD GEOMETRY IN SYMMETRIC SPACES

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1. Introduction

The classical local invariants of a submanifold in a space form are the first fundamental form, the shape operators and the induced normal connection, and they determine the submanifold up to ambient isometry. One of the main topics in differential geometry is to study the relation between the local invariants and the global geometry and topology of submanifolds. Many remarkable results have been developed for submanifolds in space forms whose local invariants satisfy certain natural conditions. The study of focal points of a submanifold in an arbitrary Riemannian manifold arises from the Morse theory of the energy functional on the space of paths in the Riemannian manifold joining a fixed point to the submanifold. The Morse index theorem relates the geometry of a submanifold to the topology of this path space. The focal structure is intimately related to the local invariants of the submanifold. In the case of space forms one can go backwards and reconstruct the local invariants from the focal structure, so it is not too surprising that most of the structure theory of submanifolds can be reformulated in terms of their focal structure. What *is* perhaps surprising is a fact that became increasingly evident to the authors from their individual and joint research over the past decade: while extending the theory of submanifolds to ambient spaces more general than space-forms proves quite difficult if one tries to use the same approach as for the space forms, at least for symmetric spaces it has proved possible to develop an elegant theory based on focal structure that reduces to the classical theory in the case of space forms. This paper is an ex-

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