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GEOMETRY OF THE ENDS OF THE MODULI SPACE OF ANTI-SELF-DUAL CONNECTIONS

PAUL M. N. FEEHAN

1. Introduction

Let X_0 be a closed, oriented, C^∞ four-manifold and let $M_{X_0,P}(g_0)$ be the moduli space of g_0-anti-self-dual connections on a principal G bundle P over X_0. The subspace $M^*_{X_0,P}(g_0)$, obtained by excluding the reducible connections is then a finite-dimensional, usually non-compact, C^∞ manifold. The moduli space $M^*_{X_0,P}(g_0)$ is naturally endowed with a metric $\mathbf{g}$ of Weil-Petersson type, called the L^2 metric, and our purpose in this article is to study the geometry of the moduli space ends.

(a) Main results. It has been conjectured by D. Groisser and T. Parker in [13], [14] and by S. K. Donaldson in [5] that the moduli space of anti-self-dual connections, endowed with the L^2 metric, has finite volume and diameter. The goal of this article is to prove this conjecture under the hypotheses described below.

Theorem 1.1. *Let X_0 be a closed, connected, oriented, simply-connected, C^∞ four-manifold with generic metric g_0 and let P be a principal G bundle over X_0 such that either (1) $G = \mathrm{SU}(2)$ or $\mathrm{SO}(3)$ and $b^+(X_0) = 0$, or (2) $G = \mathrm{SO}(3)$ and $w_2(P) \neq 0$, where $w_2(P)$ is the second Stiefel-Whitney class of P. Then the moduli space $M^*_{X_0,P}(g_0)$ of irreducible g_0-anti-self-dual connections on P has finite volume and diameter with respect to the L^2 metric $\mathbf{g}$ defined by g_0.*

We plan to discuss the case of $G = \mathrm{SU}(2)$ and $b^+(X_0) > 0$ in a subsequent article. Note that when $G = \mathrm{SO}(3)$ and $w_2(P) \neq 0$, the trivial (product) connection Θ does not appear in the Uhlenbeck compactification $\overline{M}^u_{X,P}(g_0)$. By 'diameter' we mean the sum of the diameters

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