J.DIFFERENTIAL GEOMETRY
Vol. 42, No.2 September, 1995

ON THE MODULI SPACE OF POLYGONS IN THE EUCLIDEAN PLANE

MICHAEL KAPOVICH & JOHN MILLSON

Abstract

We study the topology of moduli spaces of polygons with fixed side lengths in the Euclidean plane. We establish a duality between the spaces of marked Euclidean polygons with fixed side lengths and marked convex Euclidean polygons with prescribed angles.

1. We consider the space $\mathcal{P}_n$ of all polygons with n distinguished vertices in the Euclidean plane $\mathbb{E}^2$ whose sides have nonnegative length allowing all possible degenerations of the polygons except the degeneration of the polygon to a point. Two polygons are identified if there exists an orientation preserving similarity of the complex plane $\mathbb{C} = \mathbb{E}^2$ which sends vertices of one polygon to vertices of another one. We shall denote the edges of the n-gon P by: $e_1, ..., e_n$ and vertices by $v_1, ..., v_n$ so that $\overrightarrow{e}_j = v_{j+1} - v_j$. The space $\mathcal{P}_n$ is canonically isomorphic to the complex projective space $P(H)$ where $H \subset \mathbb{C}^n$ is the hyperplane given by

$$H = \{(e_1, ..., e_n) \in \mathbb{C}^n : e_1 + ... + e_n = 0\}.$$

Therefore, the space $\mathcal{P}_n$ can be identified with $\mathbb{C}P^{n-2}$. The length of the edge e_j will be denoted by r_j. We shall assume that all polygons are normalized so that the perimeter is equal to 1.

Originally published in Volume 42, Number 1 without the figures. For the convenience of the reader, it is reprinted here with the figures.