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QUASI-CONFORMAL HARMONIC DIFFEOMORPHISM AND THE UNIVERSAL TEICHMÜLLER SPACE

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0. Introduction

The classical theory of Teichmüller spaces uses extensively the theory of quasi-conformal mappings. However, since the development of the basic results of Eells and Sampson [11], Hartman [15] (see also [3]), Schoen and Yau [27] and Sampson [25], one can now use harmonic maps to study Teichmüller spaces. See for example [10], [30] and [34]. Some important results in this direction were obtained by Wolf [34]. A starting point in the theory of [34] is the following. Using the fact that the Hopf differential of a harmonic map between two Riemann surfaces is holomorphic, it was proved [34] that the Teichmüller space T_g of a compact surface of genus $g > 1$ is homeomorphic to the space of holomorphic quadratic differentials of a fixed compact Riemann surface of the same genus. Then using the space of holomorphic quadratic differentials as the coordinate chart for the Teichmüller space T_g, one can study many important properties of T_g, see for example [34, 18]. Note that in the proof of the result of [34] mentioned above, the fact that the Teichmüller space T_g is of $(6g-6)$-dimensions was used. Later in [18], Jost gave another proof without using this fact.

The result in [34] can be put into a more general setting. In studying the relations between harmonic diffeomorphisms on the hyperbolic space $\mathbb{H}^2$ of dimension 2 and constant mean curvature cuts in the Minkowski 3-space, the second author [31, 32] was able to construct a map $\mathcal{B}$ (see §1 for definition) from the space $BQD(\mathbb{D})$ of holomor-

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