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FLAT FLOW IS MOTION BY CRYSTALLINE CURVATURE FOR CURVES WITH CRYSTALLINE ENERGIES

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Abstract

For interface energies which are crystalline, motion of curves in the plane by crystalline curvature typically coincides with their flat curvature flow.

We study the time evolution of curves $P(t)$ in the plane moving with normal velocities equal to a "weighted mean curvature" derived from an interface energy function Φ which is assumed to be *crystalline* and *even*. Our main result is that if one starts with a *good* polygonal curve $P(0)$, then two different formulations of what such evolution should mean typically coincide. (All terms in italics are defined below.)

The first formulation is that of *motion by crystalline curvature,* in which the $P(t)$'s are computed by integration of a coupled system of ordinary differential equations in the time variable t; each $P(t)$ continues to be a good polygonal curve having no more line segments than $P(0)$ has [10].

The second formulation is that of *flat Φ curvature flow,* in which discrete time approximations are obtained by solving sequences of variational problems in which all possible curves (not just polygonal ones) are in competition; the flow is obtained as a limit of such discrete time approximations [4].

We show below that these two different procedures produce the same curve evolutions, except perhaps in cases in which many edges coalesce into one in the motion by crystalline curvature. A more painstaking

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