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SYMPLECTIC PACKING CONSTRUCTIONS

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1. Introduction

Let V^{2n} be a symplectic manifold. A symplectic k-packing of V via equal balls consists of k symplectic embeddings of a $2n$-dimensional ball with disjoint images in the interior of V. If $\operatorname{Vol} V < \infty$, there is an upper bound to the radii of the balls which admit a symplectic k-packing since symplectic embeddings preserve volume. Some natural questions include: *For fixed k, what is the least upper bound for r such that there exists a symplectic packing via k embeddings of a ball of radius r? For which k is there a full packing, i.e., for which k can the volume of the image of the packing get arbitrarily close to the volume of V?*

Using his technique of pseudo-holomorphic curves, Gromov calculated that a packing of the 4-dimensional ball of radius 1, $B^4(1)$, via 2, 3, or 4 symplectic embeddings of a closed ball does not exist if $r \geq \sqrt{1/2}$ and that a packing via 5 or 6 embeddings cannot exist if $r \geq \sqrt{2/5}$, [2 (0.3.B)]. McDuff and Polterovich, in [6], combined the pseudo-holomorphic curve theory with the theory of symplectic blow ups and proved that a packing of $B^4(1)$ does not exist for 7 embeddings when $r \geq \sqrt{3/8}$ nor for 8 embeddings when $r \geq \sqrt{6/17}$. Moreover, they proved that these obstructions are sharp: there exist packings of $B^4(1)$ via $2, 3, 4, 5, 6, 7, 8$ symplectic embeddings of a closed ball of radius arbitrarily close to $\sqrt{1/2}$, $\sqrt{1/2}$, $\sqrt{1/2}$, $\sqrt{2/5}$, $\sqrt{2/5}$, $\sqrt{3/8}$, $\sqrt{6/17}$, respectively. For higher dimensional balls, Gromov calculated that a packing of $B^{2n}(1)$ via $k \leq 2^n$ embeddings cannot exist if $r \geq \sqrt{1/2}$. McDuff and Polterovich proved that for $k \leq 2^n$, there exists a packing

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