

EMBEDDED SURFACES AND THE STRUCTURE OF DONALDSON'S POLYNOMIAL INVARIANTS

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1. Introduction

Over the past decade, beginning with the fundamental work of Donaldson, it has become ever more clear that many aspects of the differential topology of smooth 4-manifolds can be seen as developing from a rich analogy with the complex geometry of algebraic surfaces. The canonical class, the first Chern class of the complex cotangent bundle, plays a fundamental role in the study of complex surfaces because it is the simplest invariant that one can obtain from the complex structure. The canonical class determines the genus of all smooth complex curves C in a complex surface X in terms of their homology classes, through the adjunction formula:

$$(1.1) \quad 2 \text{ genus}(C) - 2 = C \cdot C + K_X \cdot C.$$

In this paper, we strengthen the analogy between the smooth and complex geometries by showing that for a large class of 4-manifolds (the manifolds of *simple type* as defined below), the smooth structure alone determines a collection of two-dimensional cohomology classes. These 'basic classes' of the 4-manifold constrain the genus of smoothly embedded 2-manifolds by an inequality analogous to the adjunction formula. If the 4-manifold is also an algebraic surface, the canonical class is often one of the basic classes, and in many cases the inequality for the genus of embedded 2-manifolds establishes, for example, that a smooth algebraic curve achieves the smallest possible genus amongst all embedded 2-manifolds in its homology class.

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