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SINGULARITIES OF THE ANALYTIC TORSION

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1. Introduction

A construction, invented by D.B. Ray and I.M. Singer [23], [24], uses zeta-functions of Laplacians to assign to an elliptic complex (equipped with an inner product) a positive real number ρ, called *analytic torsion.* Ray and Singer themselves considered the analytic torsion for the De Rham and Dolbeaut complexes. It was A.S.Schwarz [26] who first studied the analytic torsion for general elliptic complexes. It is clear that the theory of analytic torsion in this generality has potentially a very wide field of possible applications in algebraic geometry, complex analysis and in mathematical physics.

A remarkable theorem, which was conjectured by Ray and Singer [23] and then proven later by J.Cheeger [6] and W.Müller [19], states that in the case of a De Rham complex, twisted by an orthogonal representation, the analytic torsion coincides with the classical Reidemeister-Franz-De Rham torsion, constructed using "finite" information on the manifold (namely, its cell decomposition). A more general theorem relating analytic torsion of the De Rham complex to the R-torsion was found recently by J.-M.Bismut and W.Zhang [5].

Suppose now, that the original elliptic complex is being deformed; this means that the differential operators, forming the complex, vary with a parameter t, where $t \in (a, b)$. Then the analytic torsion $\rho(t)$ will be a function of the parameter t. Even if the deformation of the differentials is analytic, the torsion $\rho(t)$ will in general have singularities (zeros and poles). The nature of these singularities is related to changes in the cohomology. In fact, the beautiful geometrical picture of the analytic torsion, suggested by D.Quillen [22] (cf. also [3]), con-

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