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INFINITESIMAL RIGIDITY FOR HYPERBOLIC ACTIONS

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1. Statement of results

Let Γ be a finitely-generated group, M a compact manifold and $\varphi\colon \Gamma \times M \to M$ a C^1-action of Γ on M.

Let $\mathcal{R}(\Gamma, Diff^1(M))$ denote the variety of representations of Γ into $Diff^1(M)$. It is a natural problem to study the local structure of $\mathcal{R}(\Gamma, Diff^1(M))$ in a neighborhood of a given action.

For example, there is a natural "formal tangent space" at the point $[\varphi]$ determined by the action φ, which is given by the 1-cocycles over φ with coefficients in the continuous vector fields on M (cf. Chapter 2, [15]). The 1-coboundaries form a closed subspace of the formal tangent space, and when these two spaces are equal the action is said to be *infinitesimally rigid.*

Every action φ can be perturbed by conjugating it with a diffeomorphism of M, and the set of these conjugates yields a subvariety of $\mathcal{R}(\Gamma, Diff^1(M))$. The action φ is *C^1-locally rigid* if the set of conjugates forms an open neighborhood around $[\varphi]$ – that is, every action φ_1 which is C^1-close to φ on a set of generators of Γ must be C^1-conjugate to φ.

Weil proved that for a representation $\rho\colon \Gamma \to G$ into a connected Lie group G, infinitesimal rigidity implies rigidity, for the tangent space to the variety of representations is contained in the space of 1-cocycles over the Adjoint representation [20]. The converse is not true: there are rigid representations which are not infinitesimally rigid (cf. proof of Theorem B, [17]; section 2, [15]). For non-isometric group actions on manifolds, only partial results are known connecting infinitesimal rigidity and local rigidity (cf. [1]).

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