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QUASIGEODESIC ANOSOV FLOWS AND HOMOTOPIC PROPERTIES OF FLOW LINES

SÉRGIO R. FENLEY

Abstract

A nonsingular flow is quasigeodesic when all flow lines are efficient in measuring distances in relative homotopy classes. We analyze quasigeodesic Anosov flows in 3-manifolds which have negatively curved fundamental group. We prove that the lifts of the stable and unstable foliations to the universal cover are foliations with branching, that is, they have non-Hausdorff leaf space. Furthermore any branching is associated to freely homotopic closed orbits of the flow in the manifold and there are finitely many such branching leaves up to covering translations. Using this we prove that the limit sets of the stable and unstable leaves in the universal cover cannot be Jordan curves nor the whole sphere. Identifications of ideal points of leaves are also described using freely homotopic orbits. Finally, for any Anosov flow in such manifolds, we prove the existence of uncountably many (infinitely many of which are closed) K-quasigeodesic orbits for K big enough. The key tool is the analysis of freely homotopic closed orbits, which are completely characterized for general Anosov flows.

1. Introduction

The primary goal of this article is to study metric properties of flow lines of Anosov flows in closed 3-manifolds. The two classical families in dimension 3, namely suspensions of Anosov diffeomorphisms of the two-dimensional torus (briefly suspensions) and geodesic flows on the unit tangent bundle of surfaces of negative curvature (geodesic flows), have the following property: in the appropriate metrics, the flow lines are geodesic. Since the manifolds are compact, the flow lines cannot be minimal geodesics in the usual sense. But they are minimal in relative homotopy classes, which is the same as being minimal geodesics when lifted to the universal cover.

A natural question is to decide which Anosov flows have this metric property. The requirement that flow lines be minimal geodesics is too strong and depends on the metric. Therefore relax this to the quasi-

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