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ON MODULAR INVARIANCE AND RIGIDITY THEOREMS

KEFENG LIU

1. Introduction

Let M be a compact smooth manifold with group action, and P be an elliptic operator on M which commutes with the action. Then the kernel and cokernel of P are representations of the action group. For an element g in the action group, the Lefschetz number of P at g is

$$F(g) = \mathrm{tr}_g \mathrm{Ker} P - \mathrm{tr}_g \mathrm{Coker} P.$$

We say that P is rigid with respect to this group action, if $F(g)$ is independent of g. In the following, we will only consider S^1-action, in which case two well-known rigid elliptic operators are the signature and the Dirac operator. Obviously, if P is rigid with respect to S^1-action, then it is rigid with respect to any compact connected Lie group action.

Motivated by the work of Landweber and Stong [19], in [34] Witten derived a series of elliptic operators from LM, the loop space of M. The indices of these operators are the signature, $\hat{\mathfrak{A}}$-genus or the Euler characteristic of LM. He also derived some elliptic operators which do not have finite-dimensional analogues. The cohomological aspects of these operators were discussed in detail by Lerche, Nilsson, Schellekens, and Warner [20] and many other physicists. Surprisingly the elliptic genus of Landweber and Stong turns out to be the index of one of these elliptic operators. Motivated by physics, Witten conjectured that these elliptic operators should be rigid with respect to S^1-action. These conjectures generalize the rigidity of the usual signature, Euler characteristic and Dirac operator to infinite-dimensional manifolds.

After some partial work of Ochanine [28] and Landweber and Stong [19], these remarkable conjectures were first proved by Taubes [33], by Bott and Taubes [8]. Hirzebruch [12] and Krichever [16] proved Witten's

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