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SOME REMARKS ON THE KRONHEIMER-MROWKA CLASSES OF ALGEBRAIC SURFACES

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1. Introduction

Recently Kronheimer and Mrowka have announced a very interesting result, which sheds new light on the Donaldson polynomials [8]. They find recurrence relations between the Donaldson polynomials, and a relation between the polynomials and the minimal genus of a smooth real surface representing a homology class. To be more precise we need a definition.

For a simply connected 4-manifold X with odd $b_+ \geq 3$, we denote the SU(2) polynomials on $H_0(X) \oplus H_2(X)$ by $q_k(X)$. X is said to be of *simple type* if

$$q_k(X)(\mathrm{pt}, \mathrm{pt}, -) = 4q_{k-1}(X), \qquad d = 4k - \tfrac{3}{2}(1 + b_+).$$

For 4-manifolds of simple type it is convenient to label the polynomials by their degree on $H_2(X)$, i.e., we define

$$q_d(X) = \begin{cases} q_k(X)|_{H_2(X)} & \text{if } d = 4k - \frac{3}{2}(1 + b_+), \\ \frac{1}{2}q_k(X)(pt, -)|_{H_2(X)} & \text{if } d = 4k - 2 - \frac{3}{2}(1 + b_+), \\ 0 & \text{otherwise}. \end{cases}$$

The *Donaldson series* is then the formal power series $q(X) = \sum_d q_d(X)/d!$.

Theorem 1 (Kronheimer-Mrowka). *For every simply connected 4-manifold Xof simple type there exist Kronheimer-Mrowka classes $K_1, \ldots, K_p \in H^2(X)$ and nonzero rational numbers $a_1, \ldots, a_p$ such that*

(i) $q(X) = e^{Q/2} \sum_{i=1}^{p} a_i e^{K_i}$,
(ii) $K_i \equiv w_2(X) \pmod 2$, *for all* $i = 1, \ldots, p$,
(iii) *if* $K_i \in \{K_1, \ldots, K_p\}$, *then* $-K_i \in \{K_1, \ldots, K_p\}$,

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