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BROWNIAN MOTION ON ANOSOV FOLIATIONS AND MANIFOLDS OF NEGATIVE CURVATURE

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Abstract

We study ergodic properties of Anosov foliations. Some rigidity results are obtained, including applications to manifolds of negative curvature, and an integral formula for topological entropy. We also show that the function $c(x)$ in Margulis's asymptotic formula $c(x) = \lim_{R\to\infty} e^{-hR} \cdot S(x, R)$ is almost always not constant. In dimension 2, $c(x)$ is a constant function if and only if the manifold has constant negative curvature. Generally, if the Ledrappier-Patterson-Sullivan measure is a flip invariant, then $c(x)$ is constant. Our entropy formula yields an upper bound of Gromov's simplicial volume in terms of scalar curvature.

0. Introduction

We generalize Lucy Garnett's ergodic theory for C^3 foliations to foliations $\mathscr{F}$ of class $C^3_{\mathscr{F}}$ (for definition see §1.1), and apply it to study the ergodic properties of Anosov foliations. In §1.3 we prove

Theorem 1. *The horocycle foliations (W^{su} or W^{ss}) of a C^3-transitive Anosov system with leafwise Riemannian metric of class C^3_i ($i = su, ss$) are uniquely ergodic (i.e., they have precisely one harmonic measure).*

Then we generalize the integral formulas in [27] to Anosov foliations to obtain the following rigidity result:

Theorem 2. *For an Anosov system with its unique harmonic measure w^{ss}, the following properties are equivalent:*

1° *w^{ss} is an invariant measure of the Anosov system.*

2° *J^{ss}_t is constant along W^{ss}-leaves.*

We apply the above theory to the geodesic flow on a compact Riemannian manifold M of negative curvature. We give an explicit description of the harmonic measure w^{ss} as the weak limit of the normalized spherical measure of geodesic balls. This settles a problem raised by Katok. We also derive two formulas for the topological entropy.

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