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EQUIVARIANT IMMERSIONS AND QUILLEN METRICS

JEAN-MICHEL BISMUT

Abstract

The purpose of this paper is to construct Quillen metrics on the equivariant determinant of the cohomology of a holomorphic vector bundle with respect to the action of a compact group G. We calculate the behaviour of the equivariant Quillen metric by immersions, and thus extend a formula of Bismut-Lebeau to the equivariant case.

Let $i: Y \to X$ be an embedding of compact complex manifolds. Let η be a holomorphic vector bundle on X, and let

$$(0.1) \qquad (\xi, v) : 0 \to \xi_m \underset{v}{\to} \xi_{m-1} \to \cdots \to \xi_0 \to 0$$

be a holomorphic chain complex of vector bundles on X, which, together with a restriction map $r : \xi_{0|Y} \to \eta$, provides a resolution of the sheaf $i_* \mathscr{O}_Y(\eta)$.

Let $\lambda(\xi)$, $\lambda(\eta)$ be the complex lines which are the inverses of the determinants of the cohomology of ξ, η, i.e.,

$$(0.2) \qquad \lambda(\xi) = (\det H(X, \xi))^{-1}, \qquad \lambda(\eta) = (\det H(Y, \eta))^{-1}.$$

Let G be a compact Lie group acting holomorphically on X and preserving Y, whose action lifts holomorphically to (ξ, v) and η. Let $\widehat{G}$ be the set of equivalence classes of complex irreducible representations of G. Then we have the isotypical splittings

$$(0.3) \qquad \begin{aligned} H(X, \xi) &= \bigoplus_{W \in \widehat{G}} \mathrm{Hom}_G(W, H(X, \xi)) \otimes W, \\ H(Y, \eta) &= \bigoplus_{W \in \widehat{G}} \mathrm{Hom}_G(W, H(Y, \eta)) \otimes W. \end{aligned}$$

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