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PINCHING BELOW $\frac{1}{4}$, INJECTIVITY RADIUS, AND CONJUGATE RADIUS

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Dedicated to S. S. Chern and W. Klingenberg

ABSTRACT. The injectivity radius of any simply connected, even dimensional Riemannian manifold M^n with positive sectional curvature equals its conjugate radius. So far the corresponding result in odd dimensions has only been known under the additional hypothesis that M^n is weakly $\frac{1}{4}$-pinched. Moreover, some famous examples due to M. Berger show that the statement is even false, unless M^n is at least $\frac{1}{9}$-pinched. It has been a longstanding problem whether the pinching constant can be pushed below $\frac{1}{4}$ for odd dimensional manifolds or not. In this paper we prove that this is indeed possible. The pinching constant $\delta \in [\frac{1}{9}, \frac{1}{4})$ that is needed in our main theorem does not depend on the dimension. As an application we obtain a sphere theorem for simply connected, odd dimensional, δ_n-pinched manifolds where the pinching constant δ_n is strictly less than $\frac{1}{4}$ and up to now still depends on the dimension.

1. Introduction

By the Theorem of Bonnet and Myers any complete Riemannian manifold (M^n, g) with sectional curvature $K_M \geq \lambda^2 > 0$ is compact. Its injectivity radius $\operatorname{inj} M^n$ is bounded from above by its conjugate radius ϱ_c. If M^n is even dimensional and simply connected, equality holds. This has been shown by Klingenberg [16] using Synge's Lemma in combination with a lifting argument:

$$\operatorname{inj} M^n = \varrho_c \geq \pi/\sqrt{\max K_M}. \tag{1}$$

We are going to concentrate on the *odd dimensional case* which is much more subtle.

By the Long Homotopy Lemma in Klingenberg's proof of the Sphere Theorem [17], the injectivity radius and the conjugate radius are still equal, provided that M^n is simply connected and $\frac{1}{4}\max K_M < \min K_M$. This

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