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SESHADRI CONSTANTS, GONALITY OF SPACE CURVES, AND RESTRICTION OF STABLE BUNDLES

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1. Introduction

There exist many situations in algebraic geometry where the extrinsic geometry of a variety is reflected in clear restrictions in the way that it can map to projective spaces. For example, it is well-known that the gonality of a smooth plane curve C of degree d is $d-1$, and that every minimal pencil has the form $\mathcal{O}_C(H-P)$, where H denotes the hyperplane class and $P \in C$.

In fact, there are to date various statements of this kind concerning the existence of morphisms from a divisor to $\mathbf{P}^1$. The first general results in this direction are due to Sommese [37] and Serrano [55]. Reider [34] then showed that at least part of Serrano's results for surfaces can be obtained by use of vector bundle methods based on the Bogomolov-Gieseker inequality for semistable vector bundles on a surface.

In [3], a generalization of these methods to higher dimensional varieties is used to obtain the following statement:

Theorem 1.1. *Let X be a smooth projective n-fold, and let $Y \subset X$ be a reduced irreducible divisor. If $n \geq 3$ assume that Y is ample, and if $n = 2$ assume that $Y^2 > 0$ (so that in particular it is at least nef). Let $\phi: Y \to \mathbf{P}^1$ be a morphism, and let F denote the numerical class of a fiber.*

(i) *If $F \cdot Y^{n-2} < \sqrt{Y^n} - 1$, then there exists a morphism $\psi: X \to \mathbf{P}^1$ extending ϕ. Furthermore, the restriction $H^0(X, \psi^*\mathcal{O}_{\mathbf{P}^1}(1)) \to H^0(Y, \phi^*\mathcal{O}_{\mathbf{P}^1}(1))$ is injective. In particular, ψ is linearly normal if ϕ is.*

(ii) *If $F \cdot Y^{n-2} = \sqrt{Y^n} - 1$ and $Y^n \neq 4$, then either there exists an extension $\psi: X \to \mathbf{P}^1$ of ϕ, or else we can find an effective divisor D on X such that $(D \cdot Y^{n-1})^2 = (D^2 \cdot Y^{n-2})Y^n$ and $D \cdot Y^{n-1} = \sqrt{Y^n}$, and an inclusion $\phi^*\mathcal{O}_{\mathbf{P}^1}(1) \subset \mathcal{O}_Y(D)$.*

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