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SOME BLOWUP FORMULAS FOR SU(2) DONALDSON POLYNOMIALS

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1. Introduction

Donaldson's vanishing theorem [4] settles the question of the behavior of his polynomial invariants under the connected sum operation, provided that both summands have $b_+^2 > 0$. This leaves open the question of what happens when one adds a negative-definite manifold to a manifold which admits Donaldson invariants. Indeed, this question arises naturally in the differential topology of algebraic surfaces, since the process of "blowing up" a surface X can be realized topologically as the operation of forming the connected sum of X with $\overline{CP}^2$. (Here the bar indicates that the projective plane is given the orientation making its intersection form negative-definite.)

Let X be an oriented, simply-connected 4-manifold with $b_+^2 > 1$ and odd, and let β be some "homology orientation" for X as in [3]. These data determine a sequence of multilinear functions on $H_2(X)$, the SU(2) Donaldson polynomials, which are indexed by $k \in \mathbf{Z}$ and are denoted by $\gamma_k : S^{d(k)}(H_2(X)) \to \mathbf{Z}$. In the above expression, $S^{d(k)}$ denotes the $d(k)^{\text{th}}$ symmetric power of the vector space $H_2(X)$, and $d(k)$ is given by the relation

$$d(k) = 4k - \tfrac{3}{2}(1 + b_+^2(X)).$$

Identifying $H_2(X)$ and $H_2(\overline{CP}^2)$ with their respective images in $H_2(X\#\overline{CP}^2)$ under the natural inclusions, one can expand

$$\gamma_k(X\#\overline{CP}^2) = \sum_i \binom{d(k)}{i} \lambda_{k,i} (e^*)^i,$$

where the $\lambda_{k,i}$ are multilinear functions in $H_2(X)$, and e^* is the linear function dual to the generator e of $H_2(\overline{CP}^2)$. We have omitted the homology orientation in the above notation with the understanding that

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