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CONVEX DECOMPOSITIONS OF REAL PROJECTIVE SURFACES II: ADMISSIBLE DECOMPOSITIONS

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Dedicated to the memory of Professor D. S. Rim

Abstract

A real projective surface is a differentiable surface with an atlas of charts to real projective plane $\mathbf{RP}^2$ such that transition functions are restrictions of projective automorphisms of $\mathbf{RP}^2$. Let Σ be an orientable compact real projective surface with convex boundary and negative Euler characteristic. Then Σ uniquely decomposes along mutually disjoint imbedded closed projective geodesics into compact subsurfaces that are maximal annuli, trivial annuli, or maximal purely convex real projective surfaces. This is a positive answer to a question by Thurston and Goldman raised around 1977.

We assume that surfaces in this paper are orientable always. Let S be a real projective surface with convex boundary. We say that S is the *sum* of subsurfaces $S_1, \cdots, S_n$ if S is the union of $S_1, \cdots, S_n$, and if $S_i \cap S_j$ is the union of imbedded closed geodesics disjoint from one another or the empty set whenever i and j are integers satisfying $1 \leq i < j \leq n$ (compare with §3.1 of [14]). If S is the sum of $S_1, \cdots, S_n$, then we say that S *decomposes* into $S_1, \cdots, S_n$ (*along closed geodesics*) and that $\{S_1, \cdots, S_n\}$ is a *decomposition collection* of S. (See Appendix B of [16], and [26] for examples of this summation process.) This definition is slightly different from the one by Goldman [14] since we do not have the principal boundary conditions.

Let D be an arbitrary compact simply convex domain in a 2-dimensional sphere $\mathbf{S}^2$ such that there is a segment α and a compact smooth arc β with two common endpoints p and q such that the boundary of D δD is $\alpha \cup \beta$. The quotient projective surface of $D - \{p, q\}$ by a properly discontinuous and free action of $\langle \vartheta \rangle$ for a hyperbolic or quasi-hyperbolic projective automorphism ϑ is called a *primitive trivial annulus*. (See [5]

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