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A SIMPLE GEOMETRICAL CONSTRUCTION OF DEFORMATION QUANTIZATION

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Abstract

A construction, providing a canonical star-product associated with any symplectic connection on symplectic manifold, is considered. An action of symplectomorphisms by automorphisms of star-algebra is introduced, as well as a trace construction. Generalizations for regular Poisson manifolds and for coefficients in the bundle $\operatorname{Hom}(E, E)$ are given.

1. Introduction

A manifold M is called a Poisson manifold, if for any two functions $u, v \in C^\infty(M)$, a Poisson bracket is defined by

$$\{u, v\} = t^{ij} \frac{\partial u}{\partial x^i} \frac{\partial v}{\partial x^j}. \tag{1.1}$$

The bracket is a bilinear skew-symmetric operation, satisfying the Jacobi identity

$$\{u, \{v, w\}\} + \{v, \{w, u\}\} + \{w, \{u, v\}\} = 0.$$

An important particular case is a symplectic manifold. In this case the matrix t^{ij} has maximal rank $2n$ equal to the manifold dimension. The inverse matrix ω_{ij} defines the exterior 2-form $\omega = \frac{1}{2}\omega_{ij}\, dx^i \wedge dx^j$ which is closed in virtue of Jacobi identity.

In [1] it has been proved that, if the tensor t^{ij} has constant rank $2n > \dim M$, there exists a symplectic foliation of the manifold M, a Poisson manifold with this property being said to be regular. The leaves F of this foliation locally are symplectic manifolds, and a Poisson bracket is defined by the symplectic form ω (closed 2-form of the rank $2n = \dim F$) defined on the leaves.

In the same paper [1] the question of deformation quantization of Poisson and in particular symplectic manifolds is considered. The problem is to define an associative multiplication operation $*$, depending on parameter h (Planck constant), of two functions so that the space $C^\infty(M)$ with

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