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CONVEX DECOMPOSITIONS OF REAL PROJECTIVE SURFACES. I: π-ANNULI AND CONVEXITY

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Dedicated to the memory of Sookja Lee

Abstract

A real projective surface is a surface with a flat real projective structure. A π-annulus is an easy-to-construct real projective annulus with geodesic boundary. Let Σ be an orientable compact real projective surface with convex boundary and negative Euler characteristic. We prove that there is a π-annulus with a projective map to Σ whenever Σ is not convex.

The real projective plane $\mathbf{RP}^2$ is the quotient space of $\mathbf{R}^3 - \{O\}$ for the origin O under the equivalence relation determined by

$$\mathbf{x} \sim \mathbf{y} \quad \text{if and only if} \quad \mathbf{x} = s\mathbf{y},$$

where $s \in \mathbf{R} - \{0\}$ and $\mathbf{x}$, $\mathbf{y} \in \mathbf{R}^3$. The action of the general linear group $\mathrm{GL}(3, \mathbf{R})$ induces a transitive action of the projective general linear group $\mathrm{PGL}(3, \mathbf{R})$ on $\mathbf{RP}^2$. Felix Klein's Erlangen program states that real projective geometry is the study of properties of $\mathbf{RP}^2$ invariant under the action of $\mathrm{PGL}(3, \mathbf{R})$ (Goldman [10]). Given a differentiable surface, an atlas of charts to $\mathbf{RP}^2$ such that transition functions are restrictions of elements of $\mathrm{PGL}(3, \mathbf{R})$ induces real projective geometric properties locally and consistently on the surface from $\mathbf{RP}^2$. A maximal element of the collection of such atlases is said to be a *real projective structure.* (We omit the word *real* from the words *real projective* from now on since the projective structures that we work with are real projective structures.) A differentiable surface with a projective structure is said to be a *projective surface*, and an immersion from a projective surface to projective surface preserving projective structures is said to be a *projective* map. Let $\mathbf{RP}^2$ have the obvious projective structure (with a single coordinate chart). $\mathrm{PGL}(3, \mathbf{R})$ consists of projective automorphisms of $\mathbf{RP}^2$. The standard unit sphere

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