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DERIVATIVES OF TOPOLOGICAL ENTROPY FOR ANOSOV AND GEODESIC FLOWS

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Abstract

In the first part of this article we calculate the first and second derivatives of the topological entropy for C^∞ perturbations of Anosov flows. Of particular interest is the appearance of the "variance" (familiar from Central Limit Theorems) in our formula for the second derivative. Our proof is based on the use of symbolic dynamics and thermodynamic methods developed in [17]. In the second part of this article we consider the special case of geodesic flows, and concentrate on finding a geometric interpretation of the formula. The third and final part of the paper deals with estimates on the variance term in the formula for the second derivative.

PART ONE. ANOSOV FLOWS

Let $\phi_t : M \to M$ be a C^∞ Anosov flow on a compact manifold. Such flows include, for example, geodesic flows associated to compact manifolds with negative sectional curvatures. In a recent article, Katok, Knieper, Weiss, and the present author shows that for C^∞ Anosov flows the *topological entropy* has a C^∞ dependence on C^∞ perturbations of the flow [17]. In the present note we shall add to these results by deriving explicit formulae for the first and second derivatives.

Consider a C^∞ family of Anosov flows $\lambda \mapsto \phi^{(\lambda)}$, $\lambda \in (-\varepsilon, \varepsilon)$. Denote by

$$\lambda \mapsto \alpha^{(\lambda)} = 1 + \lambda(D_0\alpha^{(\lambda)}) + (\lambda^2/2)(D_0^2\alpha^{(\lambda)}) + \cdots$$

the *velocity change* in the structural stability theorem (i.e., the velocity change in $\phi^{(0)}$ to make it topologically conjugate to $\phi^{(\lambda)}$). We denote by

$$\lambda \mapsto h^{(\lambda)} = 1 + \lambda(D_0h^{(\lambda)}) + (\lambda^2/2)(D_0^2h^{(\lambda)}) + \cdots$$

the topological entropy $h^{(\lambda)}$ of the flow $\phi^{(\lambda)}$, and then our main result is the following:

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