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THE RIEMANNIAN STRUCTURE OF ALEXANDROV SPACES

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Abstract

Let X be an n-dimensional Alexandrov space of curvature bounded from below. We define the notion of singular point in X, and prove that the set S_X of singular points in X is of Hausdorff dimension $\leq n-1$ and that $X - S_X$ has a natural C^0-Riemannian structure.

0. Introduction

Let $\mathscr{M}(m, \kappa, D)$ denote the class of m-dimensional compact Riemannian manifolds with sectional curvature $\geq -\kappa^2$ and diameter $\leq D$. Any sequence $\{M_i\}_{i=1,2,\dots}$ of $\mathscr{M}(m, \kappa, D)$ contains a subsequence $\{M_{j(i)}\}_i$ converging to a compact metric space M_∞ with respect to the Hausdorff distance d_H (see [12]). Although we could not expect the limit space M_∞ to be a manifold, it inherits several properties of the manifolds in $\mathscr{M}(m, \kappa, D)$, i.e., M_∞ is an Alexandrov space of curvature $\geq -\kappa^2$, diameter $\leq D$, and of Hausdorff dimension $\leq m$. We say a metric space X is an Alexandrov space (of curvature bounded from below) if X is a connected, complete, and locally compact length space of curvature bounded from below and of finite Hausdorff dimension. (In [4] any such space X is called a FSCBB. The precise definition of Alexandrov space will be given in §1.) Therefore the study of Alexandrov spaces makes clear the structure of the d_H-closure of $\mathscr{M}(m, \kappa, D)$, and then it is very useful for the study of manifolds in $\mathscr{M}(m, \kappa, D)$.

Assume that X is an Alexandrov space of curvature $\geq k$. For any triple of points $p, q, r \in X$ we denote by $\tilde{\angle} pqr$ the angle at $\tilde{q}$ of a triangle $\triangle\tilde{p}\tilde{q}\tilde{r}$ in the simply connected space form of constant curvature k such that $|\tilde{p}\tilde{q}| = |pq|$, $|\tilde{q}\tilde{r}| = |qr|$, and $|\tilde{r}\tilde{p}| = |rp|$, where $|xy|$ denotes the distance between x and y. A point $p \in X$ is called an (n, δ)-*strained point* if there exist points $p_i \in X$, $i = 1, \cdots, 2n$ such that

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