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HOMOGENEOUS SUBMANIFOLDS OF HIGHER RANK AND PARALLEL MEAN CURVATURE

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Abstract

Let M^n, $n \geq 2$, be an orbit of a representation of a compact Lie group which is irreducible and full as a submanifold of the ambient space. We prove that if M admits a nontrivial (i.e., not a multiple of the position vector) locally defined parallel normal vector field, then M is (also) an orbit of the isotropy representation of a simple symmetric space. So, in particular, compact homogeneous irreducible submanifolds of the Eucildean space with parallel mean curvature (not minimal in a sphere) are characterized (and classified). The proof is geometric and related to the normal holonomy groups and the theorem of Thorbergsson.

0. Introduction

Riemannian manifolds of nonpositive curvature and submanifolds of the Euclidean space seem to be related. There are several theorems for the fist class of spaces that have a (formal) analogous result in the context of submanifolds. Their proofs seem also to have some similarities, though the concepts involved are of a quite different nature (e.g., holonomy groups of the tangent or normal connection). In the first case a very important role is played by the symmetric spaces. In the case of submanifolds this role is played by all the orbits of the isotropy representation of semisimple symmetric spaces (s-representations) (see [14]). For manifolds of nonpositive curvature with finite volume and higher rank one has the theorem of Ballmann/Burns-Spatzier [1], [2], which asserts that they are locally symmetric. On the other hand, for compact isoparametric submanifolds of higher rank one has the theorem of Thorbergsson [17] which assets that they are orbits of s-representations. The proofs of Burns-Spatzier and Thorbergsson rely on the topological Tits buildings. There is also another proof of the result of Thorbergsson in [12] which does not use Tits buildings and is related to the normal holonomy groups. (For any

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