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L^p COHOMOLOGY OF CONES AND HORNS

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Abstract

We prove the conjecture of J. Brasselet, M. Goresky, and R. MacPherson on the isomorphism between L^p cohomology and intersection cohomology for a stratified space with a Riemannian metric and conical singularities. We prove the extension of this conjecture to spaces with f-horn singularities, where $f(r)$ is any C^∞ nondecreasing function.

We study the L^p Stokes property which states that the minimal closed extension of d acting on L^p forms coincides with the maximal one. We prove that it implies the Borel-Moorse duality between the complexes of L^p forms and L^q forms. We also prove the converse for spaces with f-horn singularities under the condition that the integral $\int_0^\varepsilon f(r)^{-1}\,dr$ diverges.

1. Introduction

J. Cheeger [4] discovered that the L^2 cohomology of a compact stratified pseudomanifold with respect to a metric which has conical singularities, is isomorphic to its upper middle-perversity itersection cohomology. He showed that this is also the case for the singular metrics which he called *f-horns*; locally they are of the form $dr^2 + f(r)^2 g$ where r is the radial coordinate (i.e., the distance from the singular point), g is the metric on the link of the point, and f is a function of r of the form $f(r) = r^c$ with $c \geq 1$; in case $c = 1$ we get a cone. If L is the link, then the f-horn over it is denoted $C^f L$. (See Definitions 3.1.1 and 3.2.1.)

M. Nagase [7] considered the case $c < 1$ and showed that when L^2 cohomology is isomorphic to the intersection cohomology, although with a different perversity, greater than the middle one and dependent upon the value of c.

J. Brasselet, M. Goresky, and R. MacPherson [2] conjectured that the L^p cohomology of a metric with conical singularities is isomorphic to the intersection cohomology with a perversity $\overline{p}$ which corresponds to L^p cohomology: $\overline{p}(k) = \max\{i \in \mathbb{Z} | i < k/p\}$.

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