A HANDLE DECOMPOSITION OF AN EXOTIC $\mathbb{R}^4$

ŽARKO BIŽACA

Abstract

In [4] S. DeMichelis and M. Freedman constructed uncountably many exotic $\mathbb{R}^4$'s that can be embedded in S^4. Such an exotic $\mathbb{R}^4$ can be constructed in the boundary of a smooth compact 5-dimensional h-cobordism that is not diffeomorphic to the product cobordism. An example of nonproduct h-cobordism was explicitly described by S. Akbulut [1]. In the present article Akbulut's description is used to construct a compact handlebody which contains two copies of an eight-level Casson tower and which has the following property. Any open handlebody that is obtained from this compact handlebody by replacing each Casson tower by a Casson handle embedded in it and by removing the boundary is an exotic $\mathbb{R}^4$. A concrete example of such an exotic $\mathbb{R}^4$ is obtained by an application of the Reimbedding algorithm from [2]. This exotic $\mathbb{R}^4$ is an open handlebody fully described by an infinite link calculus picture.

0. Introduction. It is known that there are smooth, compact, simply connected h-cobordisms between nondiffeomorphic 4-dimensional closed manifolds. If $(W^5; X_0^4, X_1^4)$ is such an h-cobordism, then it clearly cannot be diffeomorphic to the product cobordism, $X_0 \times I$. However, by work of M. Freedman [6], the h-cobordism theorem holds in the category of 4-dimensional, simply connected topological manifolds, so any h-cobordism between closed, simply connected 4-manifolds is homeomorphic to the product cobordism. An analysis of Freedman's proof of the h-cobordism theorem shows that the "product structure" always smoothly exists over the complement of a flat cell in X_0 (see [6], [4], [9] or [7]). Moreover, this flat cell contains an exotic $\mathbb{R}^4$ that is embedded in both X_0 and X_1, and X_1 can be reconstructed from X_0 by changing the embedding of this exotic $\mathbb{R}^4$. A detailed exposition can be found in [4] or [9], but here we will restrict ourselves to the slightly less general situation coming from the h-cobordism in [1]. Starting with Akbulut's explicit description of this h-cobordism, we extract a handle decomposition for an exotic $\mathbb{R}^4$.