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CONTRACTION OF CONVEX HYPERSURFACES IN RIEMANNIAN SPACES

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Abstract

This paper concerns the deformation of hypersurfaces in Riemannian spaces using fully nonlinear parabolic equations defined in terms of the Weingarten curvature. It is shown that any initial hypersurface satisfying a natural convexity condition produces a solution which converges to a single point in finite time, and becomes spherical as the limit is approached. The result has topological implications including a new proof of the 1/4-pinching sphere theorem of Klingenberg, Berger, and Rauch, and a new "dented sphere theorem" which allows some negative curvature.

1. Introduction

An earlier paper by the author [1] considered a general class of fully nonlinear curvature flows of hypersurfaces in Euclidean space. In this paper we adapt these techniques to the more difficult problem of deforming hypersurfaces in Riemannian spaces. We prove that any compact hypersurface satisfying a sharp convexity condition is necessarily the boundary of an immersed disc (Theorem 1–5).

Let M^n be a smooth, connected compact manifold of dimension $n \geq 2$ without boundary, and let (N^{n+1}, g^N) be a complete smooth Riemannian manifold satisfying the following conditions:

$$-K_1 \leq \sigma^N \leq K_2, \qquad |\nabla^N R^N|_{g^N} \leq L \tag{1–1}$$

for some nonnegative constants K_1, K_2 and L. Here σ^N is any sectional curvature of N^{n+1}, ∇^N is the Levi-Civita connection corresponding to g^N, and R^N is the Riemann tensor on N^{n+1}.

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