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ON THE GEOMETRY OF ANALYTIC DISCS ATTACHED TO REAL MANIFOLDS

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Introduction

A generic manifold M is a real submanifold of the complex space $\mathbf{C}^N$ which is locally the generic intersection of real hypersurfaces. An analytic disc attached to M is an analytic mapping A from the unit disc in $\mathbf{C}$ into $\mathbf{C}^N$, continuous up to the boundary, mapping the unit circle S^1 into M. We shall say that A passes through a point $p_0 \in M$ if $A(1) = p_0$. Analytic discs have been extensively used by many mathematicians since the work of Lewy [14] and Bishop [6], and play a central role in questions of holomorphic extendibility and propagation of analyticity for Cauchy-Riemann (CR) functions defined on M. In this paper we study the geometry of the set of all small analytic discs attached to M through p_0 whose derivatives are Hölder continuous up to the boundary.

Using elementary Banach space techniques, including the implicit function theorem, we prove (see §2) that, for $p_0 \in M$, the set of discs defined as above forms an infinite-dimensional submanifold $\mathscr{A}_{p_0}(M)$ of the Banach space of all discs valued in $\mathbf{C}^N$. We also give a parametrization of $\mathscr{A}_{p_0}(M)$ as well as an explicit description of its tangent space at each disc in the manifold $\mathscr{A}_{p_0}(M)$. In particular, this allows us to construct families of discs near a given small disc without use of the Bishop equation [6].

For any $A \in \mathscr{A}_{p_0}(M)$, we consider (see §3) the analytic discs attached to $\Sigma(M)$, the conormal bundle of M, with base projection equal to the given disc A. Then, for $\zeta \in S^1$, we introduce the defect of A at ζ as the dimension of the subspace spanned by these discs in the fiber $\Sigma_{A(\zeta)}(M)$. In fact, this defect is independent of ζ if A is sufficiently small. For small discs the notion of defect given here coincides with that introduced by Tumanov [20], but ours is expressed in a geometric context and, in particular, is invariant and independent of the choice of coordinates.

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