J. DIFFERENTIAL GEOMETRY
39 (1994) 331-342

THE WEYL PROBLEM WITH NONNEGATIVE GAUSS CURVATURE

PENGFEI GUAN & YANYAN LI

1. Introduction

Weyl posed the following problem in 1916 [21]: consider the 2-sphere S^2 and suppose g^0 is a Riemannian metric on S^2 whose Gauss curvature is everywhere positive. Does there exist a global C^2 isometric embedding $X: (S^2, g^0) \to (R^3, \delta)$ where δ is the standard flat metric in a Euclidean 3-space R^3? The first attempt to solve the problem was made by Weyl himself. He suggested the continuity method and obtained a priori estimates up to the second derivatives. Later Lewy [13] solved the problem in the case of g^0 being analytic. The complete solution was given in 1953 by Nirenberg in a beautiful paper [16] under very mild hypothesis that the metric g^0 has continuous fourth derivatives. His result depends on the strong a priori estimates he had derived for uniformly elliptic equations in dimension two [17]. The result was extended to the case of continuous third derivatives of the metric by Heinz [9] in 1962. In a completely different approach to the problem, Alexandroff [1] obtained a generalized solution of Weyl's problem as a limit of polyhedra. The regularity of this generalized solution was proved by Pogorelov [18], [19].

The uniqueness question was considered by Weyl in [21]. The first proof of the uniqueness of a solution of the problem (within rigid motion and a possible reflection), i.e., a proof of the theorem that two closed isometric convex surfaces are congruent (within a reflection) was given by Cohn-Vossen [6] in 1927, under the assumption that the surfaces are analytic. It was later shortened considerably by Zhitomirsky [22]. In 1943 Herglotz [10] gave a very short proof of the uniqueness, assuming that the surfaces are three times continuously differentiable. Finally in 1962 it was extended to surfaces having merely two times continuously differentiable metrics by Sacksteder [20]. Notice that the rigidity results in [20] hold under

Received September 21, 1992 and, in revised form, January 14, 1993. The first author was partially supported by NSERC Grant OGP-0046732, and the second by NSF Grant DMS-9104293.