J. DIFFERENTIAL GEOMETRY
39 (1994) 251-280

CURVATURE MEASURES AND CHERN CLASSES OF SINGULAR VARIETIES

JOSEPH H. G. FU

The aim of the present article is to show how the approach of [8] to studying the curvature measures of a singular space yields a natural geometric treatment of the theory of Chern homology classes of singular complex analytic varieties. These classes were first considered by M. H. Schwartz [0], but were neglected at the time. Deligne and Grothendieck later introduced axioms for a conjectural theory of Chern homology classes for singular varieties. MacPherson then constructed classes fulfilling these axioms in the seminal paper [16]. Subsequent to MacPherson's work, it was shown by Brylinski, Dubson, and Kashiwara [2] that the MacPherson Chern classes of a singular variety X admit a simple expression involving the characteristic cycle of X from the theory of D-modules. Up to this point, however, a complete treatment of the properties of these classes has rested upon the somewhat indirect approach of [16]. In the meantime, we independently constructed the characteristic cycle of Kashiwara by direct geometric means [8]. The geometric insight from our construction allows us to give a direct and intuitively appealing proof of the Deligne-Grothendieck axioms, which is what we present in these pages.

The advantages of our method over that of [16] are twofold. First, the key covariance axiom of Deligne-Grothendieck for morphisms $f\colon X \to Y$ of varieties was established only indirectly for singular varieties X, using Hironaka's formidable resolution theorem. Our treatment, on the other hand, works with the singular varieties directly, without mention of resolutions. (We have, however, no proof of uniqueness for the Deligne-Grothendieck axioms apart from the original obvious argument using resolution.) Second, certain key coefficients associated to the strata of singular X are in [16] computed somewhat circuitously: viz. by initially defining a certain natural transformation T using a topological "Euler obstruction", and then *inverting* T. The Euler obstruction never enters the treatment of

Received August 20, 1991 and, in revised form, December 18, 1992. This work was supported in part by grants from the University of Georgia Research Foundation and the National Science Foundation (DMS-8902400).