J. DIFFERENTIAL GEOMETRY
39 (1994) 215-227

SYMPLECTIC TOPOLOGY ON ALGEBRAIC 3-FOLDS

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Dedicated to the memory of Andreas Floer

1. Introduction

In [18], the author introduced two Donaldson-type invariants, Φ and $\tilde{\Phi}$, based on Gromov's remarkable theory of pseudoholomorphic curves in a symplectic manifold V. Roughly speaking, ϕ is based on counting the number of holomorphic spheres in V, and $\tilde{\phi}$ is based on counting the perturbed holomorphic maps from S^2 to V. A major difference between the two invariants is that $\tilde{\Phi}$ takes into account multiple cover maps [8], but Φ does not. It turns out that $\tilde{\Phi}$ is the invariant used in topological σ models in mathematical physics. There is a remarkable mirror symmetry phenomenon among Calabi-Yau 3-folds relating this invariant to the variation of Hodge structures of its mirror. But we shall not say anything more about this phenomenon here. Instead, we refer the reader to [24], [14]. Here we deal with a different type of application, primarily for Φ. We should point out that various simple forms of these two invariants have been used by Gromov [5] and McDuff [10], [12].

Before we give the definition of Φ, recall that a symplectic manifold (V, ω) is semipositive if for any A in the image of Hurewicz map $\pi_2(V) \to H_2(V, \mathbb{Z})$, $\omega(A) > 0$ implies that $c_1(V)A \geq 0$. Now we give the definition of Φ, following the notation in [18].

Let $\Omega(V)$ be the oriented bordism group of V.

Definition. Let (V, ω) be a symplectic manifold and $A \in H_2(V, \mathbb{Z})$ with $c_1(V)A > 0$. Furthermore, if $\dim V \geq 8$, suppose that (V, ω) is semipositive. Choose a generic tamed almost complex structure J on V. For any $\alpha_1, \cdots, \alpha_k \in \Omega(V)$ such that $\deg \alpha_i \leq 2n-2$ and $\sum_i (2n - \deg \alpha_i - 2) = 2c_1(V)A + 2n - 6$, choose a representative of α_i, still denoted by α_i. We can define an integer $\Phi_{(A,J,\omega)}(\alpha_1, \cdots, \alpha_k)$ as follows. First notice the following:

Received June 24, 1992 and, in revised form, April 29, 1993. Partially supported by NSF grant DMS 9102522.