J. DIFFERENTIAL GEOMETRY
39 (1994) 173-213

CHERN-SIMONS PERTURBATION THEORY. II

SCOTT AXELROD & I. M. SINGER

Abstract

In a previous paper [2], we used superspace techniques to prove that perturbation theory (around a classical solution with no zero modes) for Chern-Simons quantum field theory on a general 3-manifold M is finite. We conjectured (and proved for the case of 2-loops) that, after adding counterterms of the expected form, the terms in the perturbation theory defined topological invariants. In this paper we prove this conjecture. Our proof uses a geometric compactification of the region on which the Feynman integrand of Feynman diagrams is smooth as well as an extension of the basic propagator of the theory.

1. Introduction

In a previous paper [2], we considered the perturbative expansion for three-dimensional Chern-Simons quantum field theory about a solution A_0 to the equations of motion. We defined what we meant by the perturbative expansion and showed perturbation theory was finite. We showed that the first term in the perturbative expansion beyond the semiclassical limit defines a geometric invariant precisely in the manner one would expect based on Witten's exact solution [10]. We conjectured and gave strong evidence that the higher terms in the expansion were geometric invariants of the same type. In this paper we prove this conjecture.

More specifically, we take A_0 to be a flat connection on a principal bundle P with a compact structure group G and a closed, oriented, three-dimensional base M. We also assume that A_0 has no zero modes, i.e., that the cohomology of the exterior derivative operator $D\colon \Omega^*(M, \mathbf{g}) \to \Omega^{*+1}(M, \mathbf{g})$, coupled to the adjoint bundle $\mathbf{g}$ of P and A_0, vanishes. By rewriting the Lorentz gauge fixed theory as a superspace theory in [2], we were able to obtain Feynman rules that could be translated succinctly into the language of differential forms. To define the gauge fixing it was necessary to choose a Riemannian metric g on M. For $l \geq 2$, the

Received April 2, 1993 and, in revised form, April 12, 1993. This work was supported in part by the Divisions of Applied Mathematics of the U. S. Department of Energy under contracts DE-FG02-88ER25065 and DE-FG02-88ER25066.