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THE SPECTRUM OF DEGENERATING HYPERBOLIC 3-MANIFOLDS

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1. Introduction

According to the cusp closing theorem of Thurston [14, §5.8], a complete, three-dimensional, noncompact manifold M, of constant negative curvature -1 and finite volume, is a limit of a sequence of compact hyperbolic manifolds $M_i \to M$. The Laplacian on the limit manifold M has continuous spectrum filling the interval $[1, \infty)$ with multiplicity equal to the number of cusps. In this paper we investigate the rate of clustering of the eigenvalues of the Laplacian on M_i as i tends to infinity. The analogous question for surfaces has been studied by Wolpert [19], Hejhal [9], and Ji [10], and a sharp estimate of the accumulation rate was obtained by Ji and Zworski [11]. In addition, Colbois and Courtois [4], [5] proved that the eigenvalues below the bottom of continuous spectrum are limits of eigenvalues of compact approximating manifolds for both Riemann surfaces and hyperbolic three-manifolds. Problems of this kind do not arise in dimensions greater than or equal to four (cf. [7]), since the number of complete hyperbolic manifolds of volume less than or equal to a given constant is finite in this case.

Suppose M has only one cusp. Then, for large i, M_i will contain a metric tubular neighborhood of a short, simple, closed geodesic γ_i of length $l_i \to 0$ and of radius $R_i \to \infty$. Let Δ_i be the Laplacian on M_i, $\operatorname{Spec}(\Delta_i)$ its spectrum and $N_i(x) = \#\{\lambda \in \operatorname{Spec}(\Delta_i) | 1 \le \lambda \le 1 + x^2\}$. Our result is that

$$N_i(x) = \frac{x}{\pi} R_i + O_x(1), \tag{1.1}$$

or equivalently (cf. (2.4))

$$N_i(x) = \frac{x}{2\pi} \log\left(\frac{1}{l_i}\right) + O_x(1). \tag{1.2}$$

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