J. DIFFERENTIAL GEOMETRY
39 (1994) 103-121

REPRESENTATIONS OF FUNDAMENTAL GROUPS WHOSE HIGGS BUNDLES ARE PULLBACKS

L. KATZARKOV & T. PANTEV

Abstract

The representations of the fundamental group of a smooth projective variety into a complex simple group are discussed in terms of the corresponding Higgs bundles. A necessary and sufficient condition is found for a representation to factor geometrically through the fundamental group of an orbicurve. The factorization question is studied further for the case of higher dimensional target varieties.

1. Introduction

The relationship between the representations of the fundamental group of a smooth projective variety and its Higgs bundles is well understood by now. In recent years the efforts of many remarkable mathematicians to put this relationship into action resulted in a new approach to the theory of the moduli spaces and lead to a major breakthrough in the long-lasting attempts for developing a nonabelian Hodge theory. The initial step belongs to Hitchin, who established this relationship in the case of algebraic curves and for representations in $SL(2, \mathbb{C})$ (see [15]). After that the relation has been studied in detail by Simpson [21], [22], [23] and Corlette [6], [8] in higher dimensions and for representations in arbitrary simple Lie group. In his work [10] Donagi developed the theory of the Hitchin maps in higher dimension and gave complete geometric description of their fibers.

Recently Simpson [23] has proven that every nonrigid Zariski dense representation of the fundamental group of a smooth projective variety in $SL(2, \mathbb{C})$ factors through the fundamental group of some orbicurve. This theorem is an analogue of a theorem of Culler and Shalen [9] and is motivated by the works of Gromov [13], Yau and Jost [17], Green and Lazarsfeld [12], Carlson and Toledo [4], and Goldman and Millson [11]. The present paper contains our attempts to investigate the same question for other simple Lie groups. Using Simpson's ideas and Donagi's theory

Received November 13, 1992.