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LINEAR HOLONOMY OF MARGULIS SPACE-TIMES

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To Aimee, with love

1. Introduction: Free discrete groups

If $\Gamma \subset \mathrm{Aff}(\mathfrak{R}^3)$ acts properly discontinuously on $\mathfrak{R}^3$, then Γ is either solvable or free up to finite index [3], [6]. If Γ is free and acts properly discontinuously on $\mathfrak{R}^3$, then Γ is conjugate to a subgroup of $\mathbf{H} = \mathbf{O}(2, 1) \ltimes \mathbf{V}$, where $\mathbf{V}$ is the group of parallel translations in $\mathbf{E} = \mathfrak{R}^{2,1}$ [3]. Let $\mathbf{G} = \mathrm{SO}(2, 1)$ and let $\mathbf{G}^o$ denote its identity component.

Complete affinely flat manifolds correspond to $\Gamma \subset \mathrm{Aff}(\mathfrak{R}^3)$ which act properly discontinuously and freely on $\mathbf{E}$. Define *Margulis space-times* as complete affinely flat 3-dimensional manifolds with free fundamental group; their existence was demonstrated by Margulis [4], [5].

Let L: $\mathrm{Aff}(\mathfrak{R}^3) \to \mathrm{GL}(n, \mathfrak{R})$ be the usual projection. If Γ acts properly discontinuously on $\mathbf{E}$, then $L(\Gamma)$ is conjugate to a free discrete group of $\mathbf{G}$; it was shown in [2].

Theorem 1. *For every Schottky group $G \subset \mathbf{G}^o$ there exists a free $\Gamma \subset \mathbf{H}$ which acts properly discontinuously on $\mathbf{E}$ and $L(\Gamma) = G$.*

$G \subset \mathbf{G}^o$ is a Schottky group if and only if all nonidentity elements are hyperbolic. The set of all Schottky groups in $\mathbf{G}^o$ is a proper subset of the set of all free discrete subgroups of $\mathbf{G}^o$. In particular, there are free discrete subgroups of $\mathbf{G}^o$, which contain parabolic elements.

We shall prove

Theorem 2. $G = L(\Gamma)$ *for some free finitely generated* $\Gamma \subset \mathrm{Aff}(\mathfrak{R}^3)$ *which acts properly discontinuously on* $\mathbf{E}$ *if and only if* G *is conjugate to a free finitely generated discrete subgroups of* $\mathbf{G}$.

For the affine manifold $\mathbf{M}$, the group of deck transformations Π acts on the universal cover $\widetilde{\mathbf{M}}$ by affine automorphisms. The developing map D: $\widetilde{\mathbf{M}} \to \mathbf{E}$ is a homeomorphism for complete $\mathbf{M}$. For every $\tau \in \Pi$ there

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