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DIVISORS ON SOME GENERIC HYPERSURFACES

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In this paper we consider generic hypersurfaces of degree at least 5 in $\mathbb{P}^3$ and especially $\mathbb{P}^4$, and reduced, irreducible, but otherwise arbitrarily singular, divisors upon them. Our purpose is to prove that such a divisor cannot admit a desingularization having numerically effective anticanonical class.

Over the past decade or so, there has been considerable interest in various questions of what might be called "generic geometry", such as the following: given a variety X which is "generic" in some sense, suppose $f: Z \to X$ is a generically finite map from a smooth variety onto some subvariety $\bar{Z} \subset X$; then what can be said about the intrinsic geometry of Z?

Perhaps the first, and still the most famous, instance of this problem concerns the case where X is a generic quintic hypersurface in $\mathbb{P}^4$. There a conjecture of Clemens [1] is (equivalent to) the statement that Z as above must have nonnegative Kodaira dimension, i.e., cannot be birationally ruled (the usual statement of Clemens' conjecture is that X should contain only finitely many rational curves of given degree, obviously equivalent to the former statement). Coming from another direction, namely Faltings' work on the Mordell conjecture, etc., S. Lang has made a series of very general conjectures which, e.g., imply in the case of a quintic 3-fold X that Z as above cannot be an elliptic fibration, if $\bar{Z} = X$.

Along similar lines, Harris has conjectured that for X a generic surface of degree $d \geq 5$ in $\mathbb{P}^3$, Z as above cannot be a rational or elliptic curve. Harris' conjecture was recently proven by G. Xu [3], who also obtains more general bounds on the genus of Z in terms of the degree of $\bar{Z}$.

Now especially from a qualitative viewpoint, one common theme to the conjectures of Clemens and Harris stands out: that is some sort of "positivity" assertion on the canonical bundle K_Z. In dimension > 1 there are of course many ways to interpret such positivity, the one involved in Clemens'

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